Dear Reader,

Some people, able both to do mathematics and to play an instrument, believe and sometimes write that there is no real link between math and music. This statement - « *I cannot see any direct, substantial connections between mathematics and music* » [1] *- makes me very uncomfortable.*

Perhaps this opinion could have been primarily conditioned by a limited and somewhat superficial experience of mathematics. The depth of the diverse facets of mathematical activity seems wanting here.

The difference between mathematics and a musical experience can be explained by the fact that mathematics is first a formalization of basic phenomena, primarily of a physical nature, and relatively rudimentary. An advanced musical expression, such that of the great composers, appears to result from purely physical acoustical factors or from some of our relatively stable perceive mechanisms, thus, susceptible to certain formalizations, musical expression can communicate an intimate sentiment, reflecting the composer's mental and emotional states, as well as the various physiological and environmental factors, past and present which have influenced him.

However, the link between mathematics and music is very strong. It stands at the level of the roots of the working of the human mind. At the present time, we do not understand its functioning. We do not understand the mechanisms of elaboration of the shape upon which the geometrical and topological intuition becomes established. We do not know how

---

[1] Christian Krattenthaler, Mathematics AND Music, Personal Views on a Difficult Relationship, EMS
1 Newsletter June 2017, 41-54.

to define and to represent the different modalities of sensitivity or intuition. At most, we can say that every form of activity of the cerebral machine more or less resonates with all the others, and consequently activates them with more or less intensity.

One of the best attestations of the presence of this link is the degree of proximity and of passion that great mathematicians have always maintained with music. These have thus marked the last century with their imprint:

- Hassler Whitney, who was initially a violinist. While in Switzerland, he followed a master class and decided to become a mathematician only after being captivated by a mathematical exposition.

- Also Henri Cartan, who hesitated between becoming a pianist or a mathematician, and in his lifetime, played regularly with Jean Dieudonné.

Apart from the moving richness of the expression of the score, the seduction that only sound quality, the architectures of chords (cf for instance http://www.mathemusic4d.net) and the score (which is sometimes locally a kind of tessellation of chords) are not unrelated to that of mathematics. I imagine that Bourbaki the elder would have voted in this direction.

After the vibration of sound, the vibration of light. I surprised a young artist - http://artmultiplevero73.free.fr we will admire her gallery of portraits, faces! - drawing a set of lines forming a network of trajectories, that associated with a conventional pursuit curve, or a plane network of trajectories coming from a degenerate node. Here then is an absolutely non-mathematical artist manifesting a geometrical sensibility, finding by herself forms defined by mathematicians. It reminds us of the late sculptor John Robinson. Also ignoring topology, he "invented" the Möbius ribbon by himself and materialized several of them. He is partially celebrated thanks to topologist Ronnie Brown. Fascinated by one of Robinson's sculptures representing the famous ribbon, he made the image of this sculpture the logo of the University of Bangor (Wales) .

What happens if you plunge your hand or any object into the phase portrait of a dynamical system? Véronique Boulesteix engraved some of her drawings on some white wood. Her first engravings use classical geometric patterns (see Figures 1, 2 and 3), and could serve the mathematical pedagogue who would like to introduce his/ her public to the theory of groups. The two original engravings (Figures 4 and 5) can also be a pleasant illustration of an introductory course to the theory of dynamical systems. The last, rather humorous, seems to be created from the trajectories of a plane dynamics converging towards a focus. Can we see in it a symbol of the mathematician working on a mathematical

theory by hand?

Whether it is mathematics, music, or visual art, the underlying mechanisms of creation obey the same rules. It is not at all strange that the most stable modes of operation emerging spontaneously and at first unconsciously on the actantial plane may end up being coded in a similar way.

Claude

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Claude Bruter, Publisher. Contributors: Véronique Boulesteix, Sharon Breit-Giraud, Richard Denner, Jos Leys. Website: http://www.math-art.eu

European Commission Interest representative