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Kinetic structures of cyclic knots and links as further development of tensegrity principle

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Abstract

The paper explores knots and links of resilient filaments and their connection with tensegrity structures. "Classical" tensegrity of compressed struts and stretched tendons derives from basic principle of woven structures and X modules, which correspond with contacting crossings of resilient knots and links.
Vertical and horizontal bending of X modules and joining together their free ends make it possible to substitute tensegrity systems of separated compressed and stretched elements with a complex resilient bended structures with pure compression in the contact points of their crossings. Such structures topologically equivalent to complicated cyclic knots and links and can transform in space taking on the shapes of different surfaces.

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Keywords: tensegrity; knots; links; borromean rings; form-finding.

1. Introduction

Nearly for a century, starting from the artworks by Russian constructivist of Latvian origin Karl Ioganson of early 1920s (Skelton, de Oliveira, 2009), the principle of tensegrity structures is continuing its successful development. The R. B. Fuller's neologism "tensegrity", which means "tensional integrity", provides wide interpretation of this

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principle in different practical applications that go far beyond the “classical” tensegrity structures made with compressed struts and stretched tendons connected together in a stable spatial configuration.

2. Tensegrity and Quasi-Tensegrity

The basic tensegrity structure is composed of two crossing struts and four tendons connecting their ends. Kenneth Snelson, who patented this structures in 1960 as basic elements for more complicated tensegrities, named them “X modules” or kite frames (Figure 1, a). Snelson emphasizes that “the simple kite frame … is a human invention and probably thousands of years old” (Snelson, 2012). He regards X modules as quasi-tensegrity because two struts contacting each other opposing them to the real minimum tensegrity module of three struts and nine tendons.

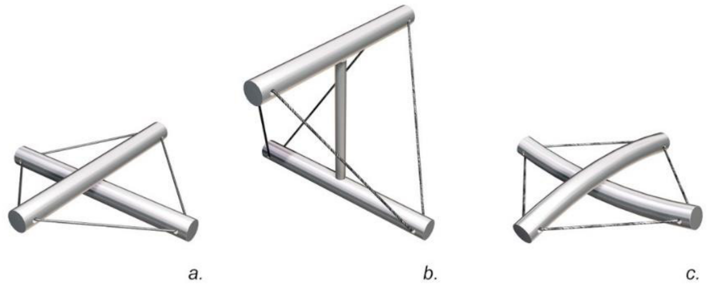

Fig. 1. X module (a), regular tetrahedron (b), bended X module (c).

The X modules as a simplest (quasi)-tensegrity structures implicitly contain different ways of their possible development. It is important to indicate that any X module topologically is a tetrahedron (Motro, 2003) — the simplex of 3d space. In order to get a geometrically regular tetrahedron with all edges of the equal length one may insert a vertical strut (a column) between the central points of the two contacting struts and connect the end points of this “double-T” configuration. If the lengths of the top and bottom struts equals to 1, the distance between their axes is $1/\sqrt{2}$, it is possible to find points at their ends so that distances between them are also equal to 1 that gives a regular tetrahedron (Figure 1, b).

The pure force of compression in this tetrahedron acts only in the central column as well as pure forces of tension act in the four tendons, but the top and bottom struts are subjected by bending forces. Exactly the same distribution of forces can be seen in an X module: the force of compression is concentrated in the point of contact of two struts. If material of the struts allows resilient deformations, the further increasing of tension in the four tendons bends the struts so that all of the tendons become lying in the same plane (Figure 1, c). In this case X module leads to an idea of “bended tensegrity” or “bow tensegrity” structures (Pars, 2016), that not contradicts to another Snelson’s definition of tensegrity as a “basic prestressed tension-compression cell” (Snelson, 2012).

At the same time, Snelson regards tensegrity structures as woven ones and identifies weaving with tensegrity. One of his early works is a 4×4 square grid composed of 16 spatially deformed (but not bended) X modules that form a woven pattern (Snelson, 2012). Analogously, to construct a piece of regular woven quasi-tensegrity tissue with pure compression in its contacting crossing points the bended X modules must be arranged in one of two basic woven patterns: orthogonal (Figure 2, a) or triangular-hexagonal.

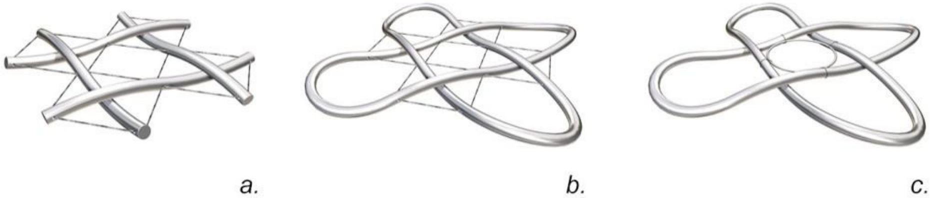

Fig. 2. Regular woven quasi-tensegrity tissue (a), struts with connected free ends as an alternating link (b),
compressing of the connected struts by a ring (c).

The structures may be prestressed much more by means of connecting together the free ends of bended struts. In this case the resilient struts transform into connected alternating knots or links (Figure 2, b). The topological connectivity of knots and links make redundant the four stretching tendons that surround every crossing. Instead of the complicated network of tendons it is sufficient to join together only central points of knots and links for example by a ring in order to keep them prestressed (Figure 2, c).

Bending and connecting of the free ends of a woven grid make the resilient struts reversibly deformed in different directions: vertically in the regions of contacting crossings and horizontally in the peripheral segments. It is obvious that these two sorts of bending can be combined in the form of doubly bended X modules by means of connecting of the ends of the resilient struts by four tendons of different sizes (Figure 3, a). The doubly bended X modules let get the base for more complicated connected woven patterns of alternating knots and links (Figure 3, b, c). The only restriction for their arrangements consists in the demand that the integral horizontal bend of all of the connected struts in the resulting knot or link necessarily has the same direction. This demand derives from the property of elastic energy of bent closed resilient struts that tends to take its minimum value and twists the knots in the space forcing them to take energetically balanced shapes (Kozlov, 2011).

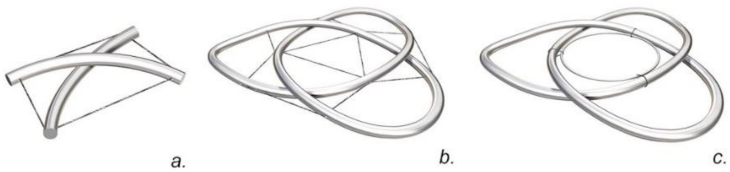

Fig. 3. Doubly bended X module (a), trefoil knot as an arrangement of three doubly bended X modules with connected free ends (b), trefoil knot reinforced by stretching ring (c).

The structures of this type maintain pure vertical compression forces in the contacting points of the crossings and distribute tension forces mixed with horizontal compression forces inside of the three-dimensionally bended closed cylindrical resilient struts. The well-known R. B. Fuller's definition of tensegrity as "*islands of compression in an ocean of tension*" (Fuller, 1962) in the case of prestressed structures of resilient knots and links may be transformed into the next one: islands of vertical (normal) compression in an ocean of 3-d bending or mixed horizontal tension and compression.

3. Bow Structures and Rings

The horizontal component of 3d bending namely the joining together of the neighboring free ends of X modules has its elementary manifestation in the joining together of two ends of the same resilient strut. If the ends are joined by a tendon the form of prestressed structure depends of relationship between the lengths of the strut and the tendon: the shorter is the tendon, the more bended and so more prestressed is the strut (Figure 4, a e). When the length of the tendon tends to zero (Figure 4, f), the composition of the strut and bended tendon can be replaced with the only strut in the shape of a ring (Figure 4, g). Ring is an utmost form of bow structures and pure horizontally bended or 2d bended element.

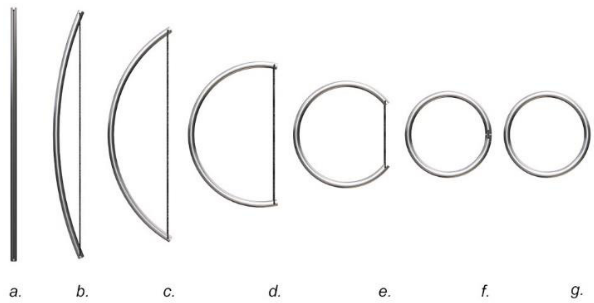

Fig. 4. Bow structures with different relationship between strut and tendon lengths (a f), prestressed ring structure (g).

Mathematics defines a knot as a one-dimensional curve placed in ordinary three-dimensional space so that it begins and ends at the same point and does not intersect itself (Neuwirth, 1979). This definition is true for a ring, and so from mathematical point of view a ring is a trivial knot. An alternating link of three interwoven rings is known as the Borromean rings. It was mathematically proved (Lindström and Zetterström, 1991) that the only horizontally bended or flat Borromean rings are impossible and in order to keep their interweaving they also must be bended in vertical direction 1. At the same time a linkage of three flat triangles that topologically equivalent to the Borromean rings is possible as a spatial structure (Jablan, 1999).

A smooth representation of the Borromean rings in 3d space may be constructed with three planar oval curves in three orthogonally related planes which function as central lines of closed surfaces with round cross-sections, namely tori. The curves may have different geometrical forms depending of the relative thickness of their cross-sections up to the most possible tight configuration (Sullivan, 2012).

Three orthogonally related great circles on a sphere represent an octahedron – their intersections coincide with the vertices of an octahedron inscribed in the sphere. The close connection between octahedron and the Borromean rings let mathematically construct 4d hyperbolic surface (3d manifolds) as compliment of the Borromean rings by means of gluing two ideal octahedra (Thurston, 1997).

If the Borromean rings are a physical structure of closed resilient struts with round cross-sections, which have a common center and placed in three orthogonally related planes, the six octahedron vertices coincide with the contacting points of their six crossings. This is an elementary model of spherical point surface materialized of six connecting crossings. It is important to emphasize that the points of contacts lay on great circles of the sphere, but the central lines of the rings have the shape of ovals and intersect with corresponding great circles in four points.

4. Transformation of the Borromean Rings

Central lines of flat projection of the Borromean rings topologically correspond with projection of an octahedron into a plane of its face (Figure 5, a). In contrast with parallel projection, this flat representation of polyhedra does not have overlapping faces (Hilbert and Cohn Vossen, 1999). Consecutive smoothing of the straight lines of the projection of an octahedron (Figure 5, b, c) make it possible to transform it into geometrically regular diagram of the Borromean rings (Figure 5, d).

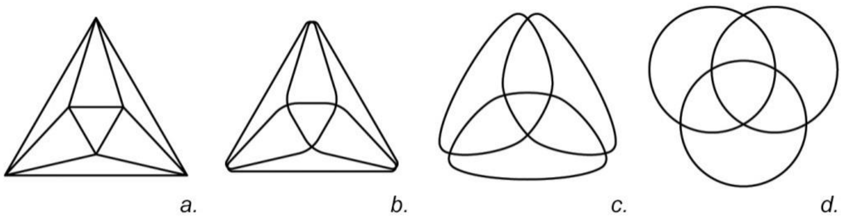

Fig. 5. Projection of octahedron into plane of its face (a), smoothing of straight lines (b, c), geometrically regular diagram of the Borromean rings (d).

Flat projection of the Borromean rings represents a particular position of horizontally and vertically bended structure of resilient rings when all of the contacting points of the crossings lay in the same plane (Figure 6, a). Topologically this structure is an octahedron, because the connections between the six vertices remain constant. This property of the resilient rings provides the reason to treat them as a form-finding structure. One can transform the rings keeping as constant the size of the central face so that the contacting points of the crossings take a spatial configuration of a spherical segment (Figure 6, b). In this case, the points of contacts lay on small circles of the sphere, and the rings remain doubly bended.

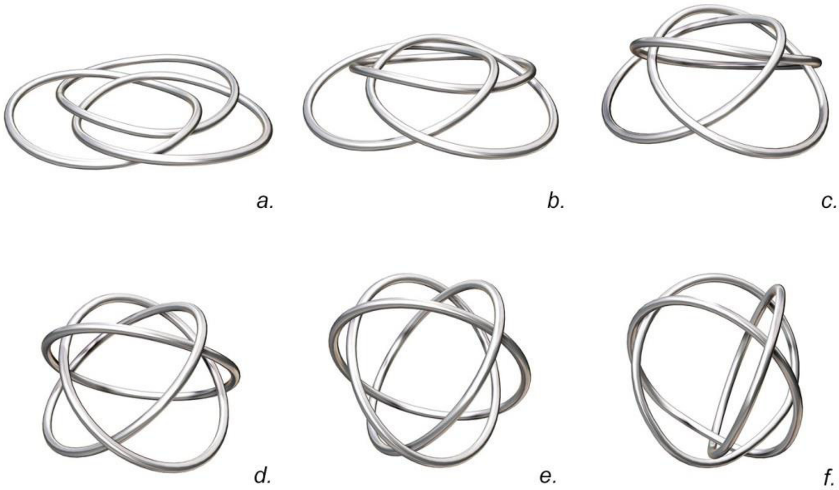

Fig. 6. Stages of transformation of a cyclic knot with fixed central face (a-f)

The distances between the contacting points of the rings change as well as change the contacting spots of the corresponding pairs of the rings. This leads to redistribution of the bending in the vertical or rather normal to the spherical surface direction. The resilient properties of the rings together with complex double bending make them to maintain physical contacts in all of their positions. The contacts move along the rings and at the same time reciprocally rotate around their central lines. In whole the resilient Borromean rings subjected to the spatial transformation behave like wave mechanisms (Dobrolyubov, 2003) transporting the contact points.

This is the most important feature of the form-finding structures of resilient cyclic links and knots. Redistribution of the bending or spatial curvature of their closed resilient components keeps the number of contacting points and the topological connections between them but changes their position in space and distances between them.

Continuation of the transformation of the Borromean rings with constant central or top face and continuously shrinking of the bottom face of the octahedron make the contacting points to form the shape of hemisphere (Figure 6, c), truncated sphere (Figure 6, d) and the sphere itself (Figure 6, e), that is the configuration of vertices of regular octahedron. In this position, the points are lying on the great circles of the sphere and the central lines of the rings are flat oval curves. Further reducing of the bottom face of the octahedron transforms the great circles back into small ones because the top and bottom faces become not of the same size (Figure 6, f).

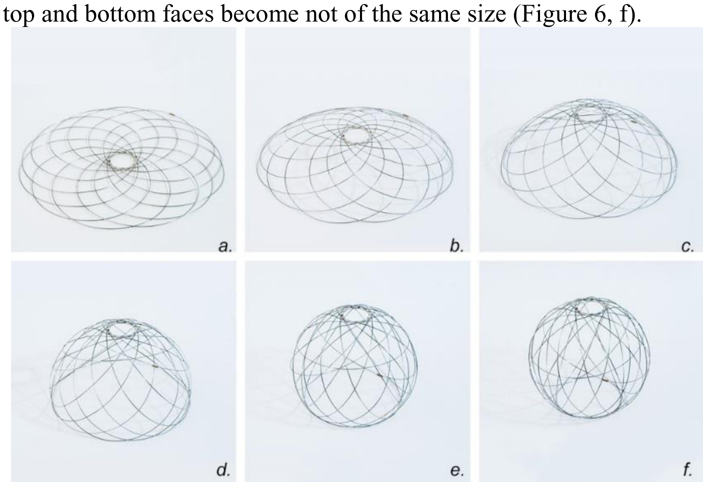

Fig. 7.

The simple example of the Borromean rings and their transformation as a whole structure gives a general picture of form generative processes typical for different type of cyclic knots and links of resilient materials (Kozlov, 2013). In practice it is much more convenient to work with knots composed of a single resilient filament than with links of rings of the same material (Figure 7, a-f). It is difficult to synchronize processes of spatial transformation for each of the rings that is important in order to receive a regular and predictable shape of an ensemble of contacting points. In its turn, a complicated cyclic knot redistributes the inner tensions between its loops and tunes itself in accordance with the outer forces of transformation. This type of complex work of resilient knots and links make them similar to tensegrity structures in the sense of synergetic behaviour of all their elements.

5. Conclusion

This is not just a coincidence. The both branches of form generation – tensegrity and transformable point surfaces, grow from the same root, namely X modules or kite frames and their spatial compositions based on the principle of weaving.

One of the branch leads to polar separation of compressed and stretched elements. In classical tensegrity, a network of stretched tendons separates all compressed struts and wholeness of the structure depends of each element. As a result, tensegrity structures find their position of minimum energy as spatial 3d objects.

The other branch leads to integration of compressing and stretching in complex resilient bending and emerging of pure compression points in the contacts of crossings. The principle of integration demands topological connectivity of the bended elements in order to keep resilient prestressing that is the product of bending. As a result, structures of transformable point surfaces find their position of minimum energy as practically 2d objects or woven surfaces.

6. Endnotes

B. Lindström and H. Zetterström finished their paper with the next idea: “It is interesting to make a model of the Borromean rings in some elastic material like thin iron wire. Try to make the “circles” as perfect as possible! The arrangement of rings will then attain infinitely many stable positions in space” (Lindström and Zetterström, 1991).

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